Mother Nature Knows Best:

Results on Wireless Ad Hoc Networks Based on Analogies with Physics

Stavros Toumpis

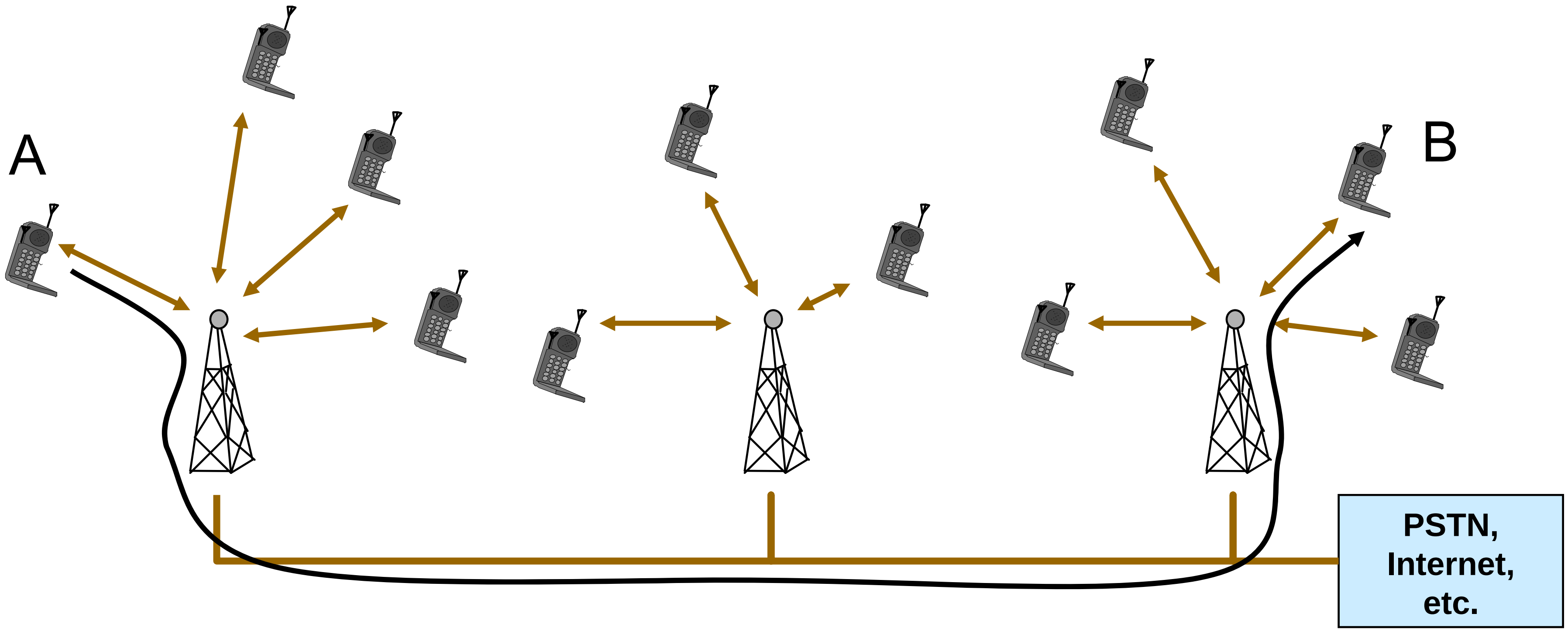
(Informatics Dept., AUEB)

(Τμήμα Πληροφορικής, ΟΠΑ!)

In This Talk

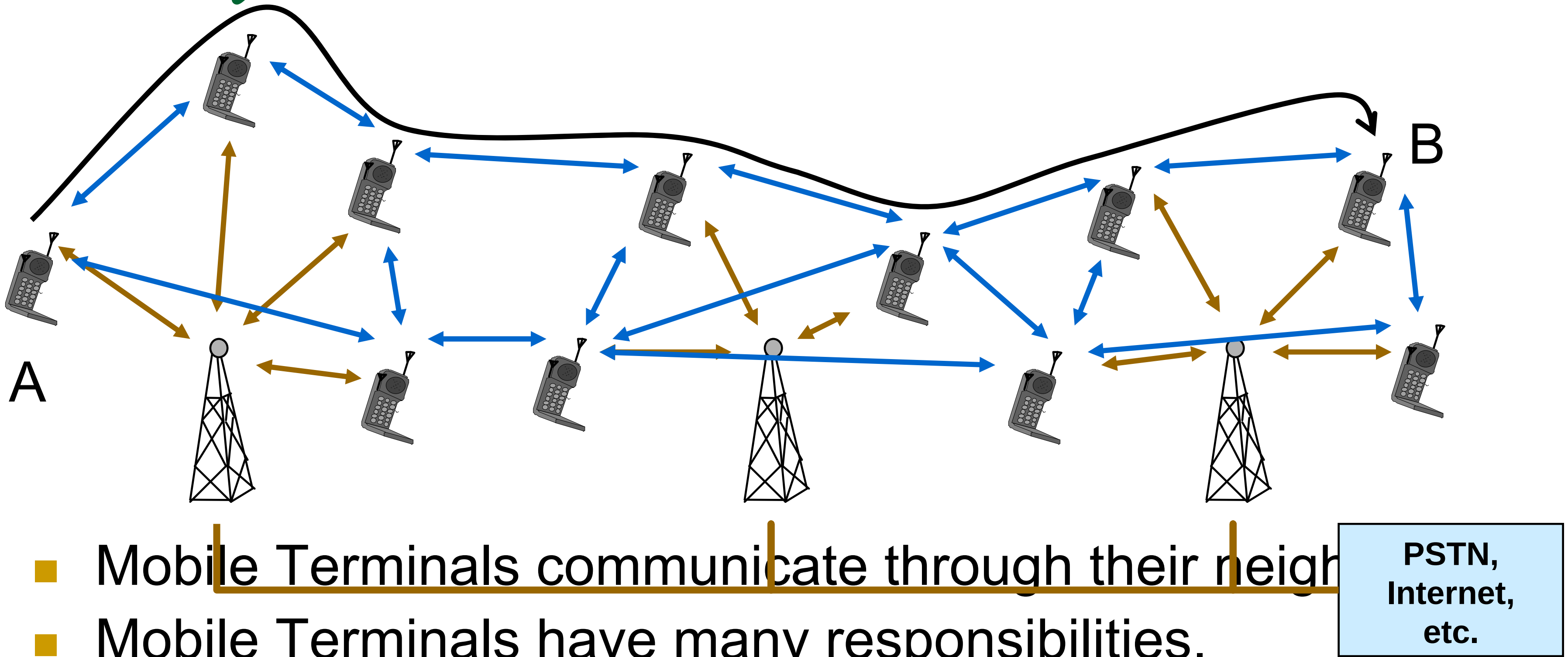
- Overview of Wireless Ad Hoc Networks
  - Applications
  - Research Challenges
- One line of research on the investigation of their fundamental properties
  - Unifying theme: analogies with physics
  - “Packetostatics”
  - “Packetoptics”
  - “Interferistors”

Cellular Wireless Networks



- Mobile terminals communicate with others exclusively through base stations.
- Mobile terminals have very little responsibility.
- A ***wireless access network***

Truly Wireless Networks



- Mobile Terminals communicate through their neigh
- Mobile Terminals have many responsibilities.
  - For example, they must forward other terminals' data.
- Much more challenging
- Known as wireless multihop networks, packet radio networks, ad hoc networks, or simply **wireless networks**

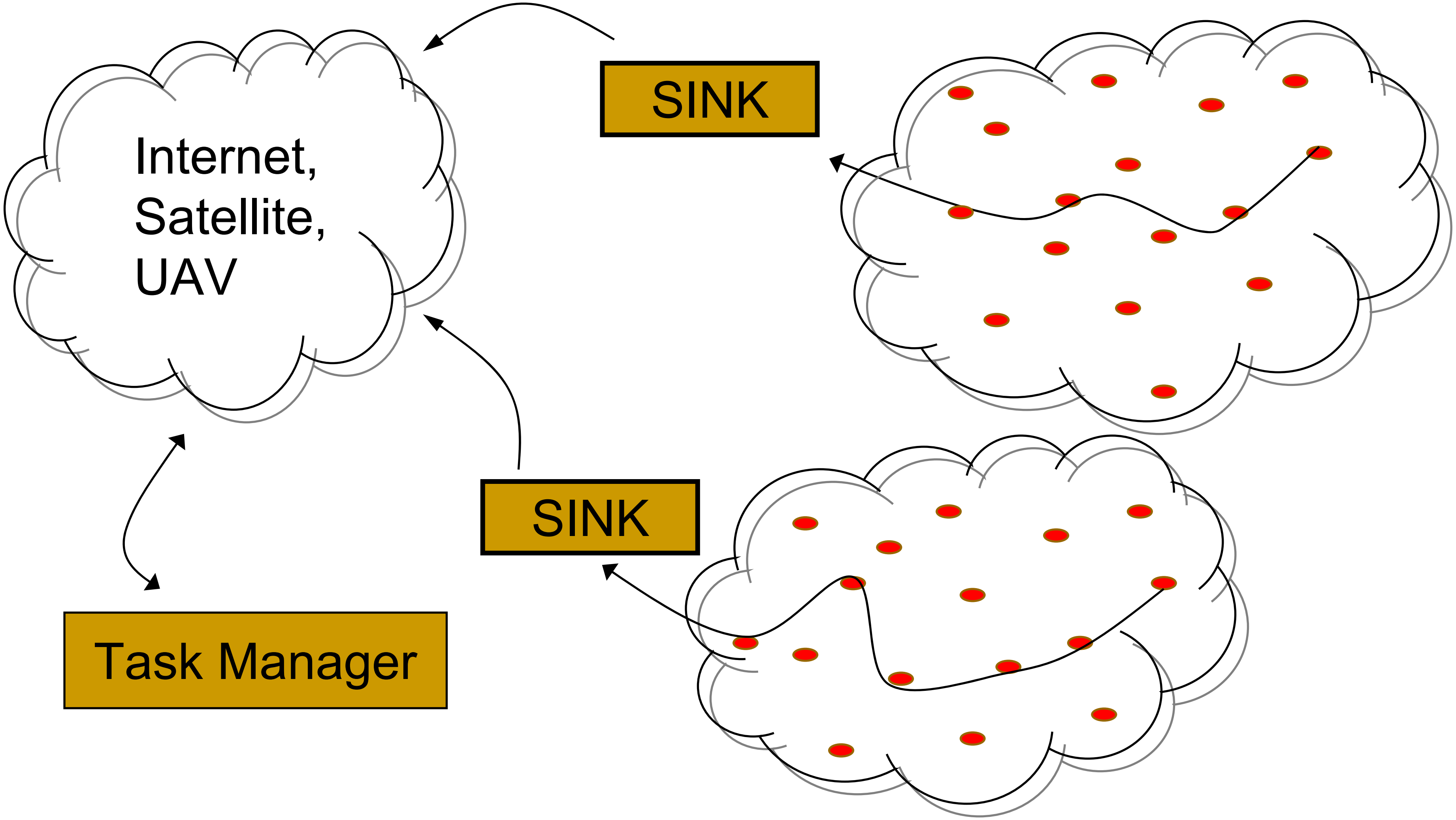
Fundamental Challenges and Advantages

- Challenges:
  - Wireless channel exhibits interference.
  - All protocols must be distributed, so that the network performance scales with the number of nodes.
  - Bandwidth is typically limited, so minimizing protocol overhead is critical.
- Advantages:
  - Fast to build (no need to set up base stations, etc.)
  - Robust (no single point of failure)
  - Good spectral efficiency

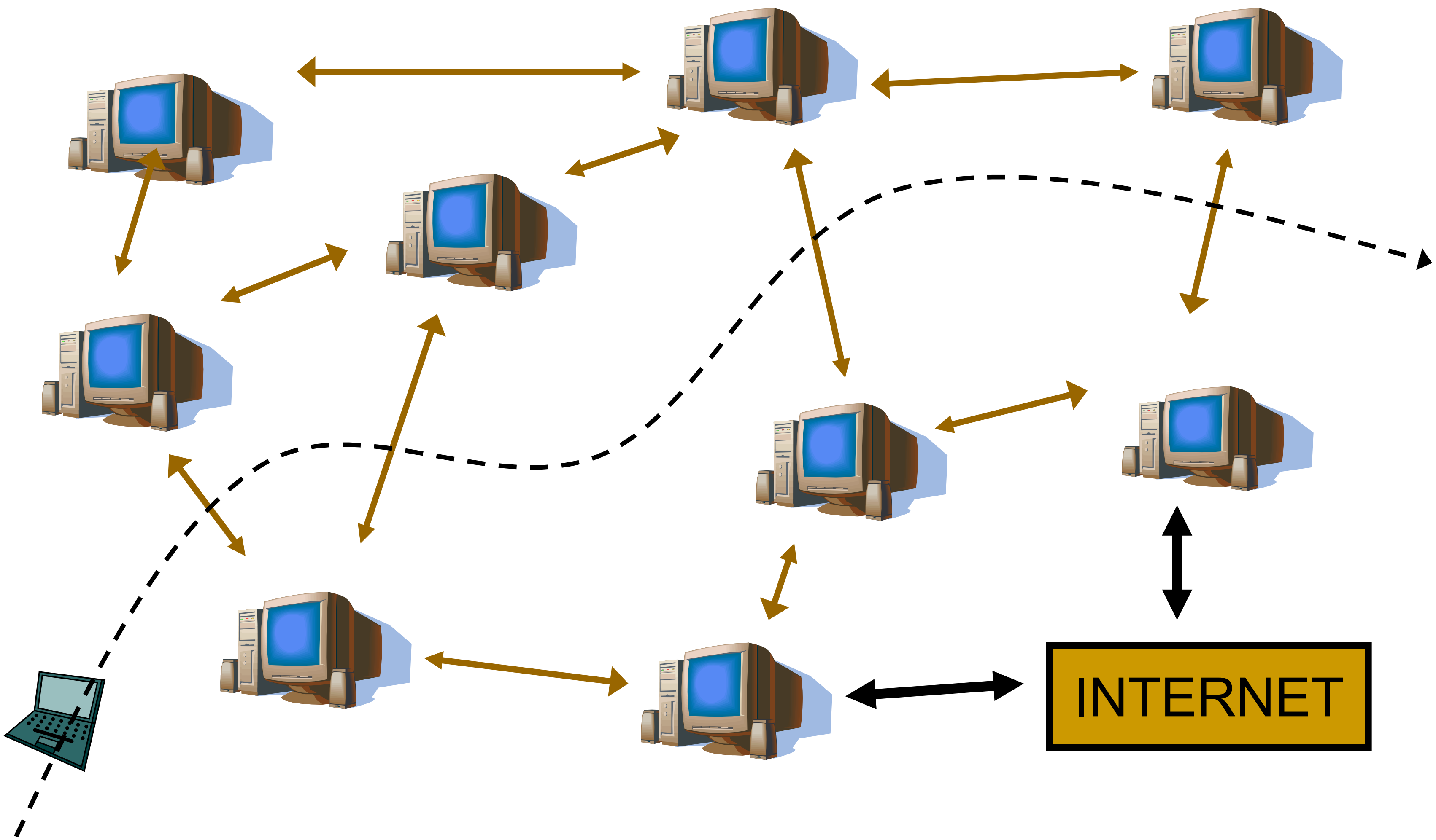
Research

- Theoretical Performance Bounds (Capacity)
  - How much throughput is achievable, with what delay, with what energy, what are the tradeoffs?
  - Mostly using idealized protocols, various branches of probability, also graph theory, brute-force intelligence, etc.
- Protocols
  - Power Control, Medium Access Control, Topology Control, Routing, Cross-layer Design
  - Mostly using simulation, probability theory, algorithmic complexity, etc.

1. Wireless Sensor Networks

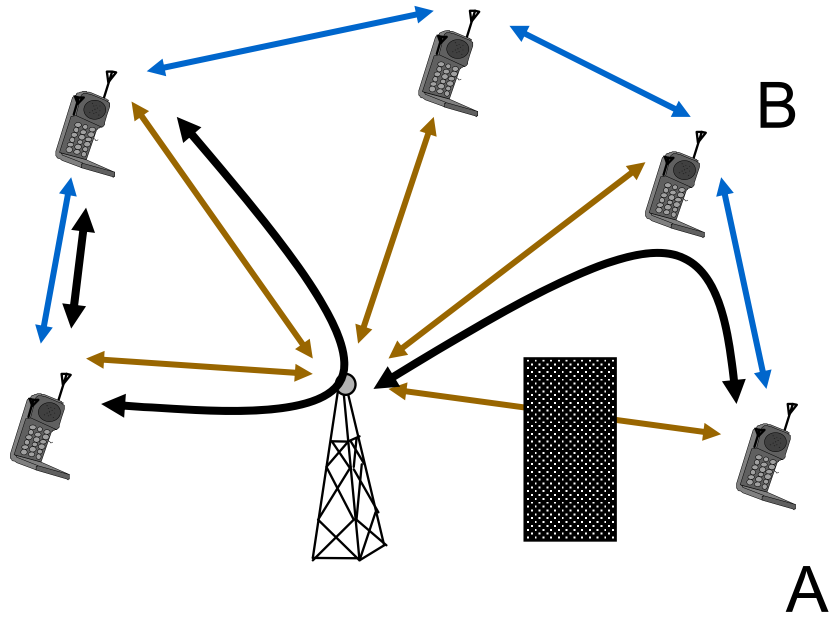


2. Wireless Mesh Networks



3. Next Generation Wireless Access (Hybrid) Networks

In next generation wireless access networks, mobile terminals will exchange information directly with each other, saving energy and bandwidth.

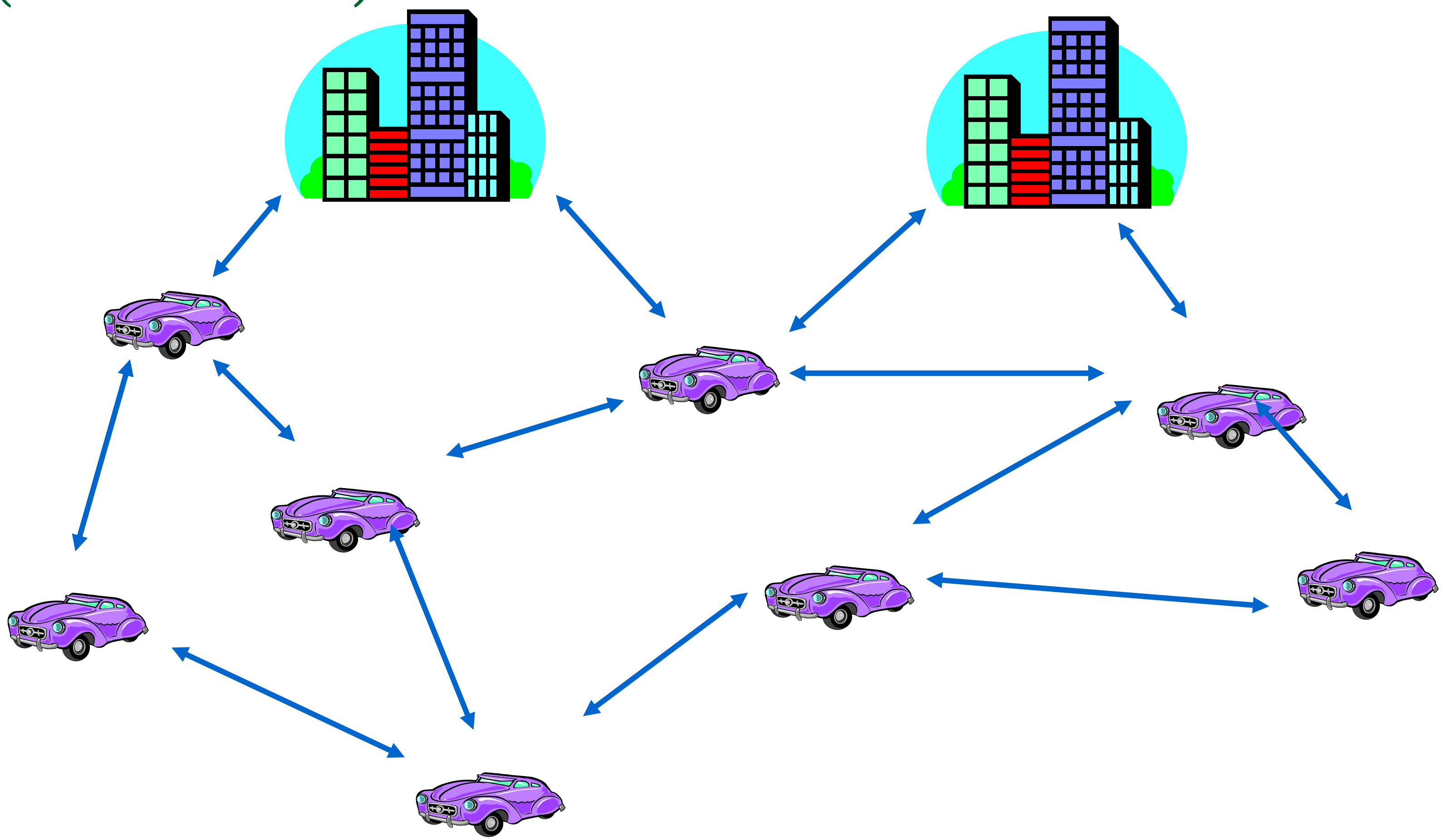


In next generation networks, user B will forward user A's data

With 3G technology, user A looses connectivity

Under current 3G technology, mobile phones only communicate with the base stations

4. Vehicular Ad Hoc Networks (VANETs)



Rest of Talk

1. "Packetostatics"
   - Optimal placement of nodes in wireless sensor networks using analogies with Electrostatics
2. "Packetoptics"
   - Optimal route design using analogies with Optics
3. "Interferistors"
   - Optimal Load Balancing using Circuit Theory

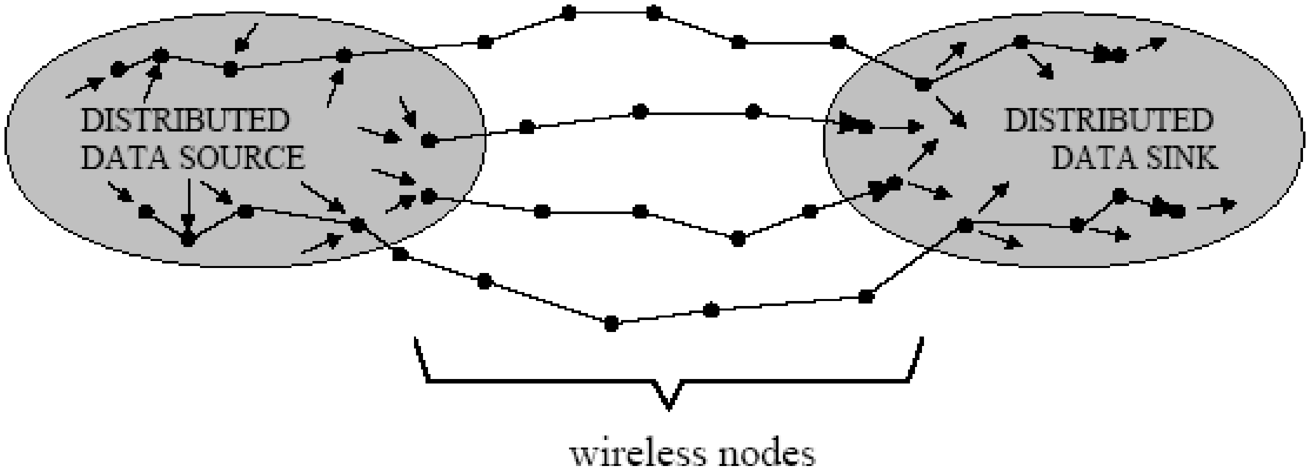
Underlying Theme

- In the modeling phase we frequently arrive at equations / tradeoffs / concepts occurring in nature
- Analogies with Physics *should* be exploited
  1. We gain intuition
  2. We end up with problems beaten to death!
- *Especially* true in wireless networks
  - Spatial component

Part A: “Packetostatics”

• S. Toumpis and L. Tassiulas, “Packetostatics: Deployment of Massively Dense Sensor Networks as an Electrostatics Problem,*” in Proc. IEEE INFOCOM* 2005, vol. 4, Miami, FL, Mar. 2005, pp. 2290-2301.
• S. Toumpis and G. A. Gupta, “Optimal Placement of Nodes in Large Sensor Networks under a General Physical Layer Model,” in *Proc. IEEE SECON 2005,* Santa Clara, CA, Sep. 2005, pp. 275-283.
• S. Toumpis and L. Tassiulas, “Optimal Deployment of Large Wireless Sensor Networks,” *IEEE Trans. on Inform.Theory*, vol. 52, no. 7, pp. 2935-2953, July 2006.

Setting



- Wireless Sensor Network:
  1. Sense the data at the source
  2. Transport the data from the sources to the sinks.
  3. Deliver the data to the sinks.
- Problem: Minimize number of nodes needed
- What is the best placement for the wireless nodes? What is the traffic flow it induces?

Macroscopic View*

- This problem is **way too complicated** to be solved without proper abstractions
- Standard approach is based on **microscopic quantities**: individual node placement, individual link properties, etc.
- We can take a novel macroscopic approach, using **macroscopic quantities**: node density, data creation density, etc.

(*P. Jacquet, "Geometry of Information Propagation in massively dense ad hoc networks," MOBIHOC 2004)

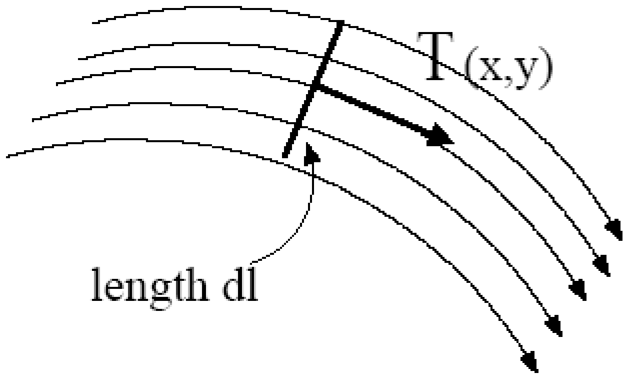
The Program

1. Define macroscopic quantities
2. Connect macroscopic quantities through **‘constitutive laws’**
3. Do analysis with macroscopic quantities

- Approach opens gateway to new (or old, depending on how you look at it) Math:
  - Calculus of Variations, Partial Differential Equations, Optics, Electrostatics, etc.
- Results are not as detailed as with standard approach, but detailed enough to remain useful

Macroscopic Quantities

- **Node Density Function** $d(x,y)$, measured in nodes/m^2.
  - In area of size dA centered at (x,y) *there are* $d(x,y)dA$ nodes
- **Information Density Function** $\rho(x,y)$, measured in bps/m^2.
  - If $\rho(x,y)>0$ (<0), information is created (absorbed) with rate ρdA over an area of size dA, centered at (x,y).
- **Traffic flow function** $\mathbf{T}(x,y)$, measured in bps/m.
  - Traffic through incremental line segment is $|\mathbf{T}(x,y)|dl$.



The Optimization Problem

- Let $$d(x, y) = G(x, y, |\mathbf{T}(x, y)|)$$

  be the density of nodes needed to support the sensing/transport/delivery
- Optimization Problem:

$$\text{minimize}: \qquad N = \int G(x, y, |\mathbf{T}(x, y)|^2) dS$$

$$\text{subject to}: \qquad \nabla \cdot \mathbf{T}(x, y) = \rho(x, y).$$

- Minimization over all possible traffic flows **T***(x,y)* that satisfy the constraint
- Standard tool for such problems: Calculus of Variations

Result

- The traffic flow is given by:

$$\mathbf{T}(x,y) = \frac{1}{2G'(x,y,H(x,y,|\nabla\varphi|)}\nabla\varphi,$$

- where the potential function *φ* is given by the scalar non-linear partial differential equation:

$$\nabla\cdot\left(\frac{\nabla\varphi}{2G'(x,y,H(x,y,|\nabla\varphi|)}\right) = \rho$$

- together with appropriate boundary conditions, and *G', H*, properly defined functions

Special Case: $d(x,y) = |\mathbf{T}(x,y)|^2$

- The traffic flow **T** and information density ρ must satisfy:

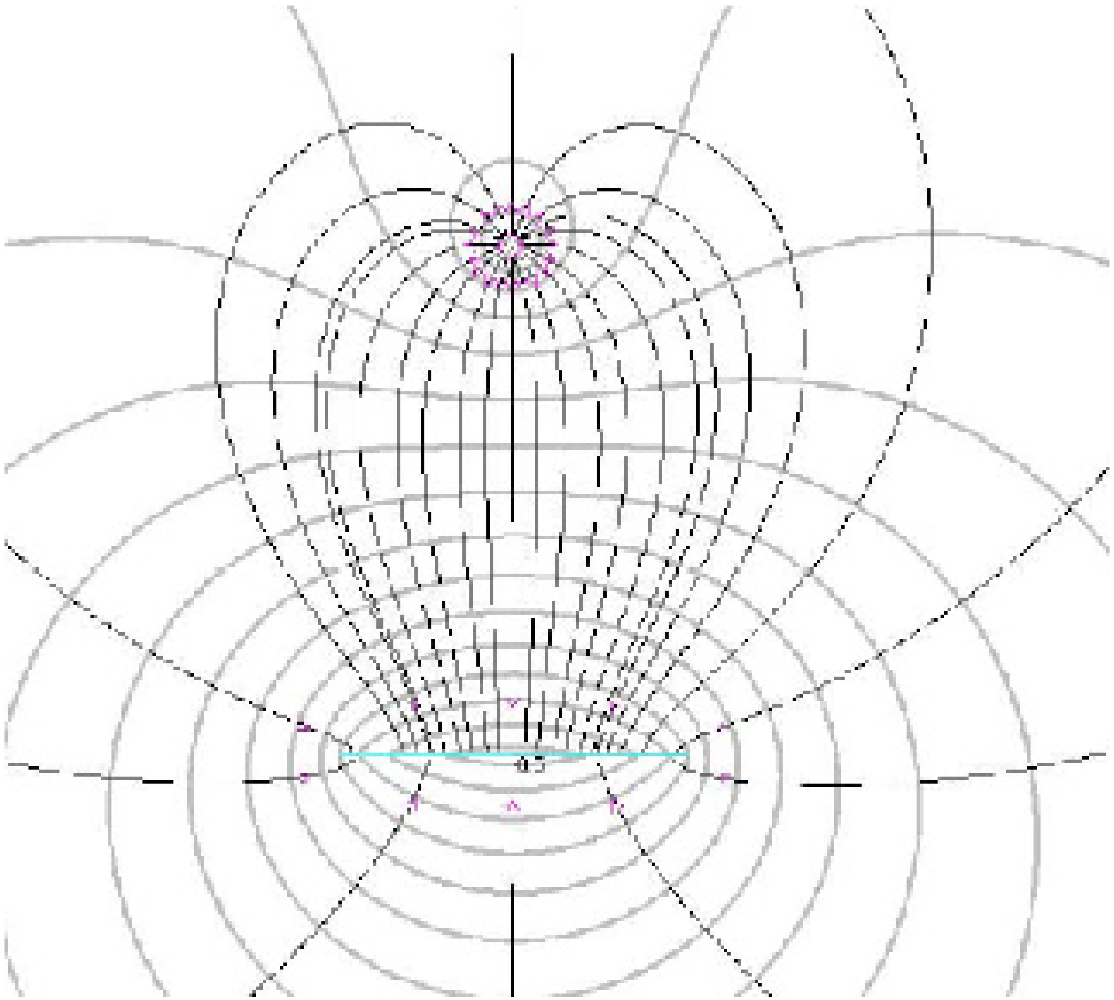
$$\nabla \cdot \mathbf{T} = \rho, \quad \nabla \times \mathbf{T} = 0.$$

- In free space, the electric field **E** and the charge density ρ are uniquely determined by:

$$\nabla \cdot \mathbf{E} = \rho, \quad \nabla \times \mathbf{E} = 0.$$

- Therefore, the optimal traffic distribution is the same with the electric field when we substitute the sources and sinks with positive and negative charges!

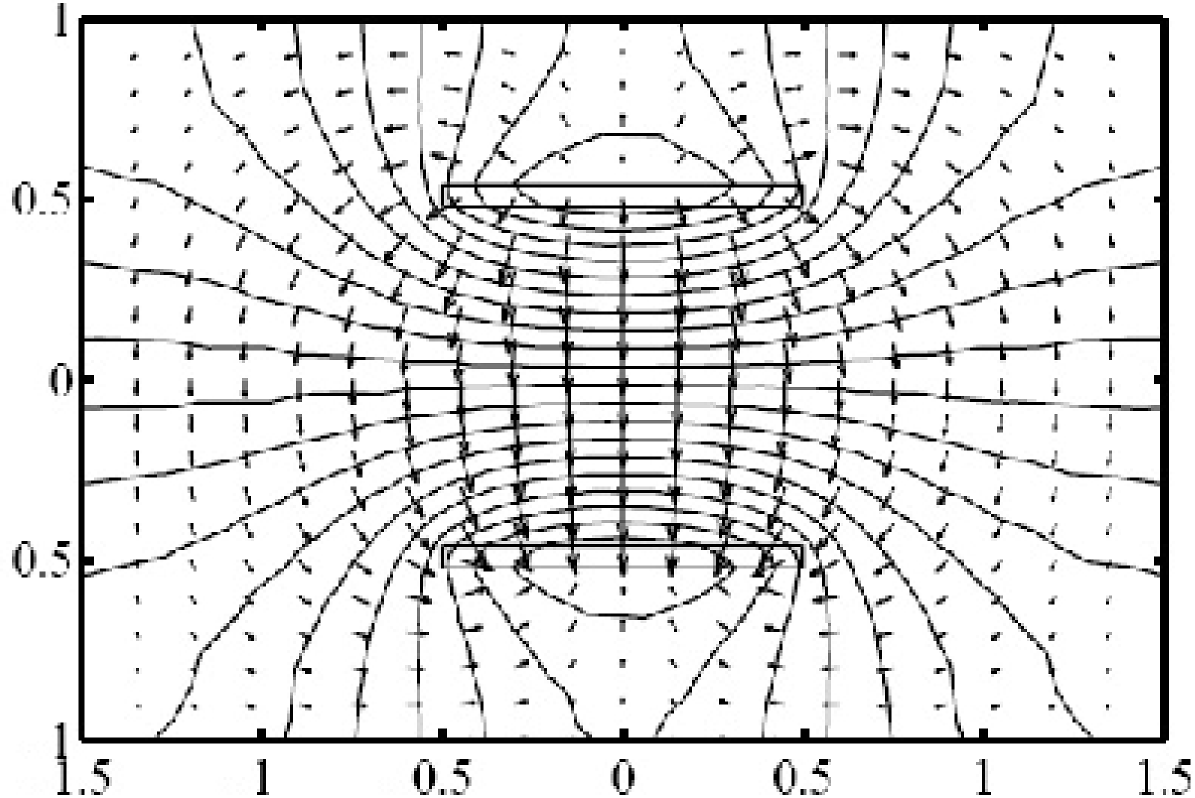
Example: A point source and a linear sink



Analogy is uncanny!

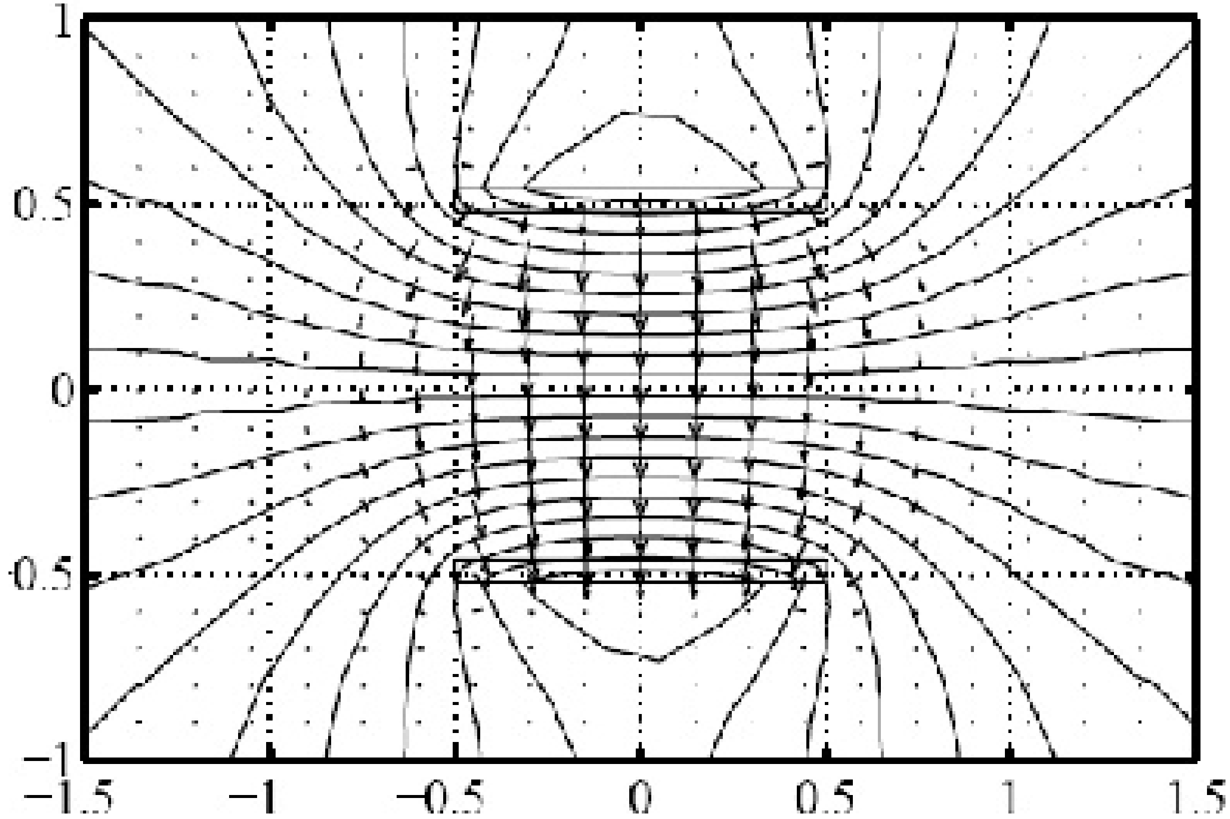
Electrostatics	Networks
Potential differences	Number of hops
Non-homogeneous dielectrics	Non-homogeneous propagation environments
Conductors	Mobile sources and sinks
Thomson’s theorem	Source/Sink placement optimization
Intersection of electric field lines and equipotential lines	Node locations

Example: Gupta/Kumar physical layer



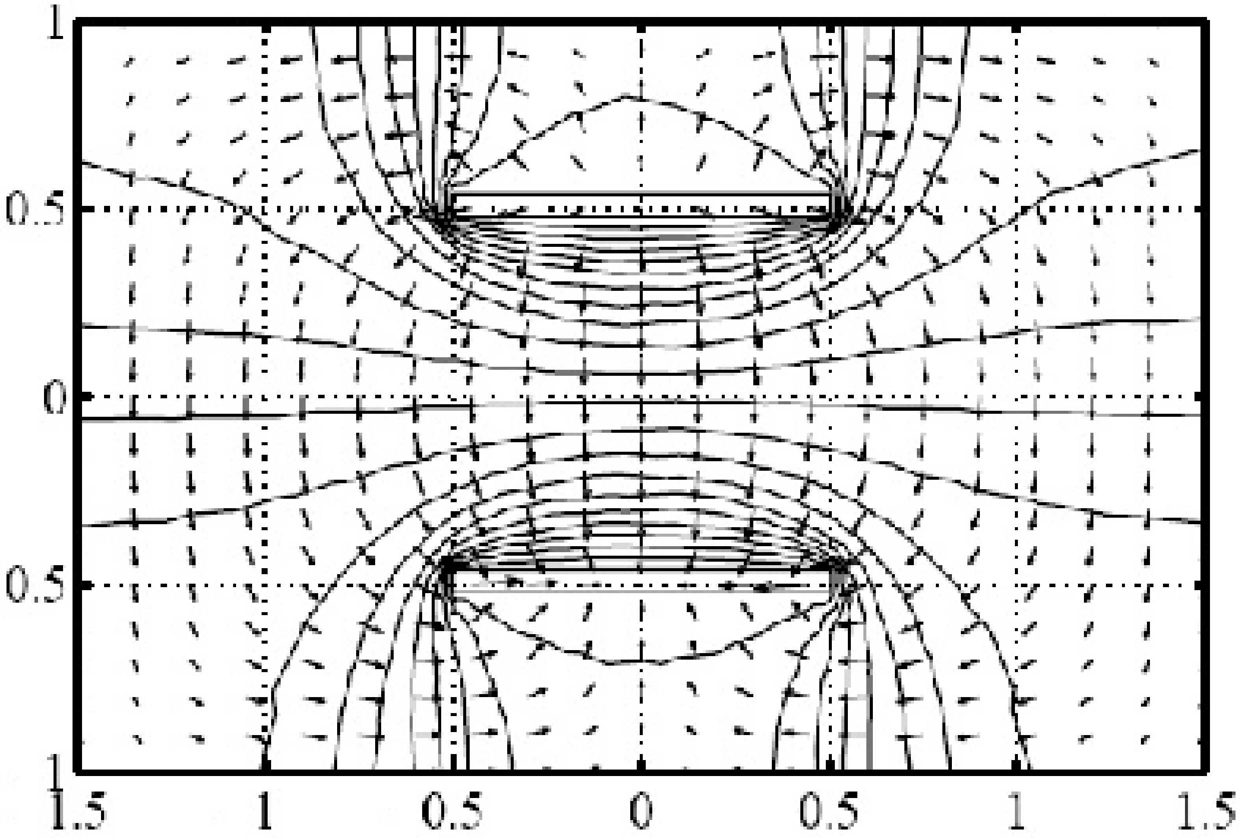
$$\left|\mathbf{T}(x,y)\right|_{\max} = c_1\left[d(x,y)\right]^{\frac{1}{2}}$$

Example: Super Gupta/Kumar



$$\left|\mathbf{T}(x,y)\right|_{\max} = c_1\left[d(x,y)\right]^{\frac{2}{3}}$$

Example: Sub Gupta/Kumar



$$\left|\mathbf{T}(x,y)\right|_{\max} = c_1\left[d(x,y)\right]^{\frac{3}{8}}$$

Part B: “Packetoptics”

- R. Catanuto, S. Toumpis, and G. Morabito, “Opti{c,m}al: Optical/Optimal Routing in Massively Dense Wireless Networks”, in *Proc. Infocom 2007*, Anchorage, AL, May 2007.
- R. Catanuto, S. Toumpis, and Giacomo Morabito, “On Asymptotically Optimal Routing in Large Wireless Networks and Geometrical Optics Analogy,” to appear in the Computer Networks journal.

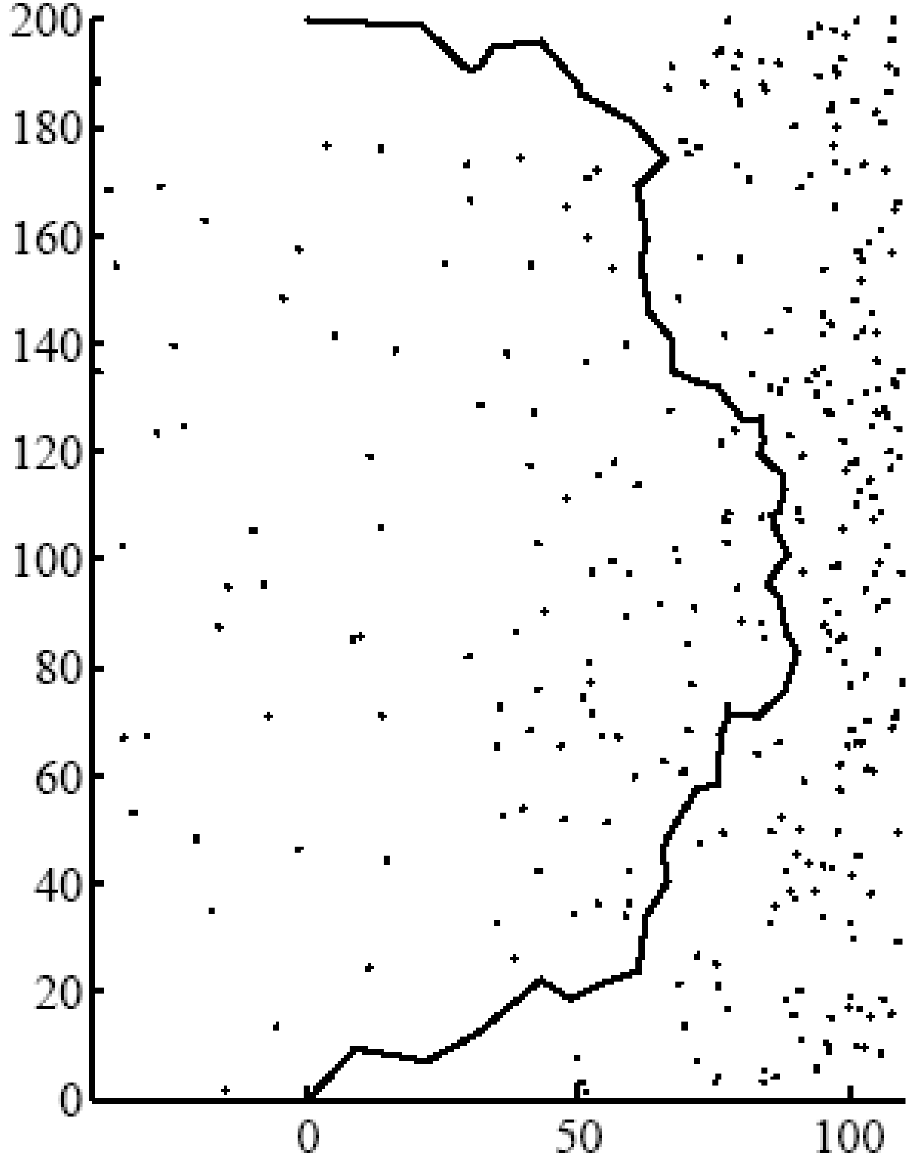
Appetizer

- Problem: Find route between (0,0) and (0,200) with minimum cost.
- Nodes distributed according to spatial Poisson process

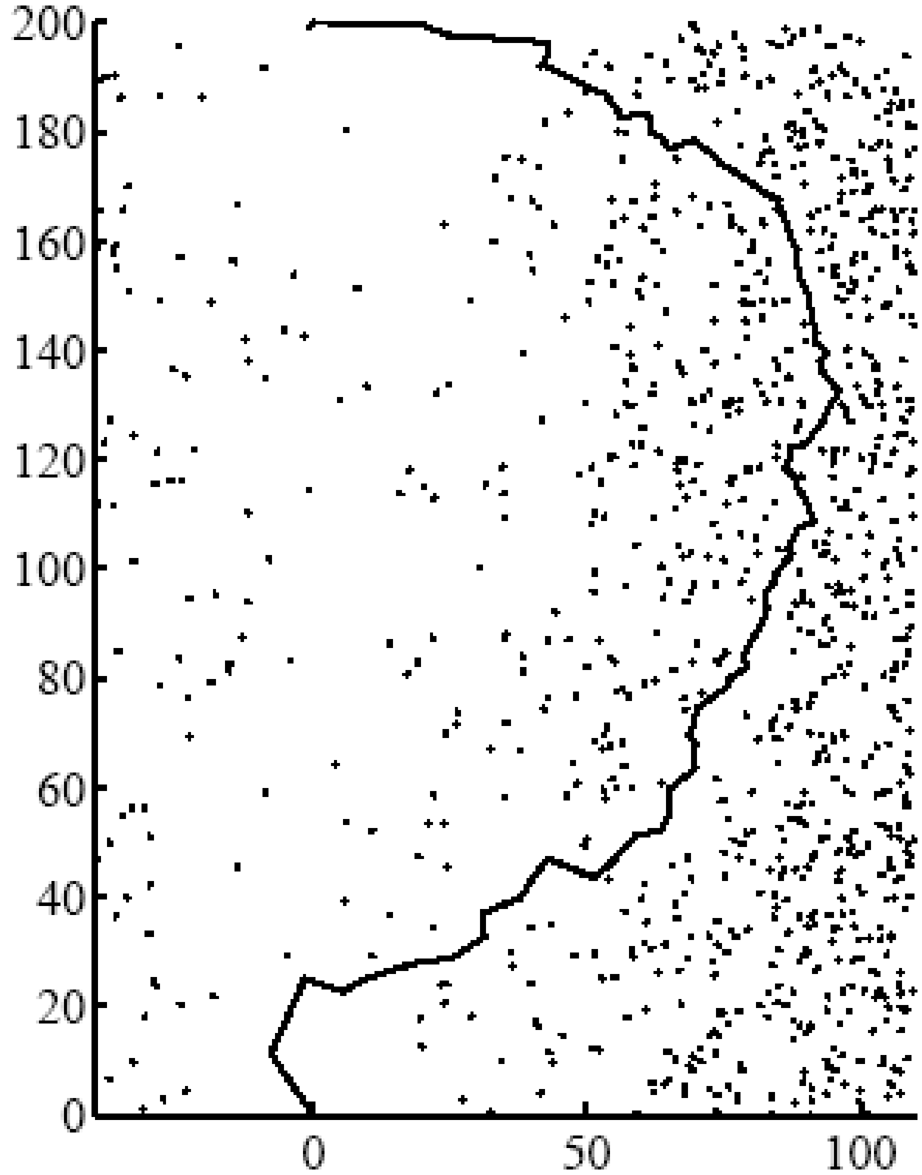
$$\lambda(x,y) = \frac{1}{30}\left(10^{-4}x^2 + 0.05\right)$$

- Cost per hop increases quadratically with hop length:

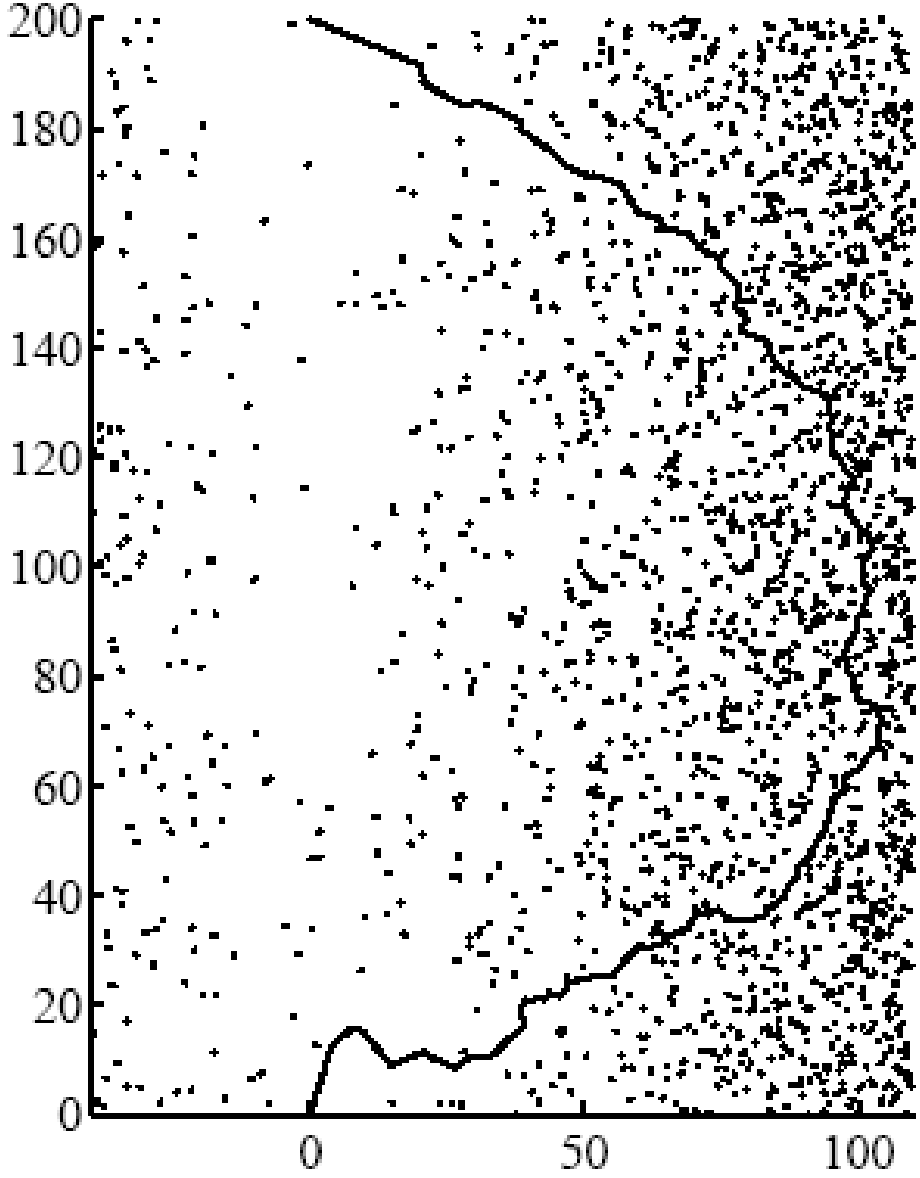
$$c(d) = ad^2.$$



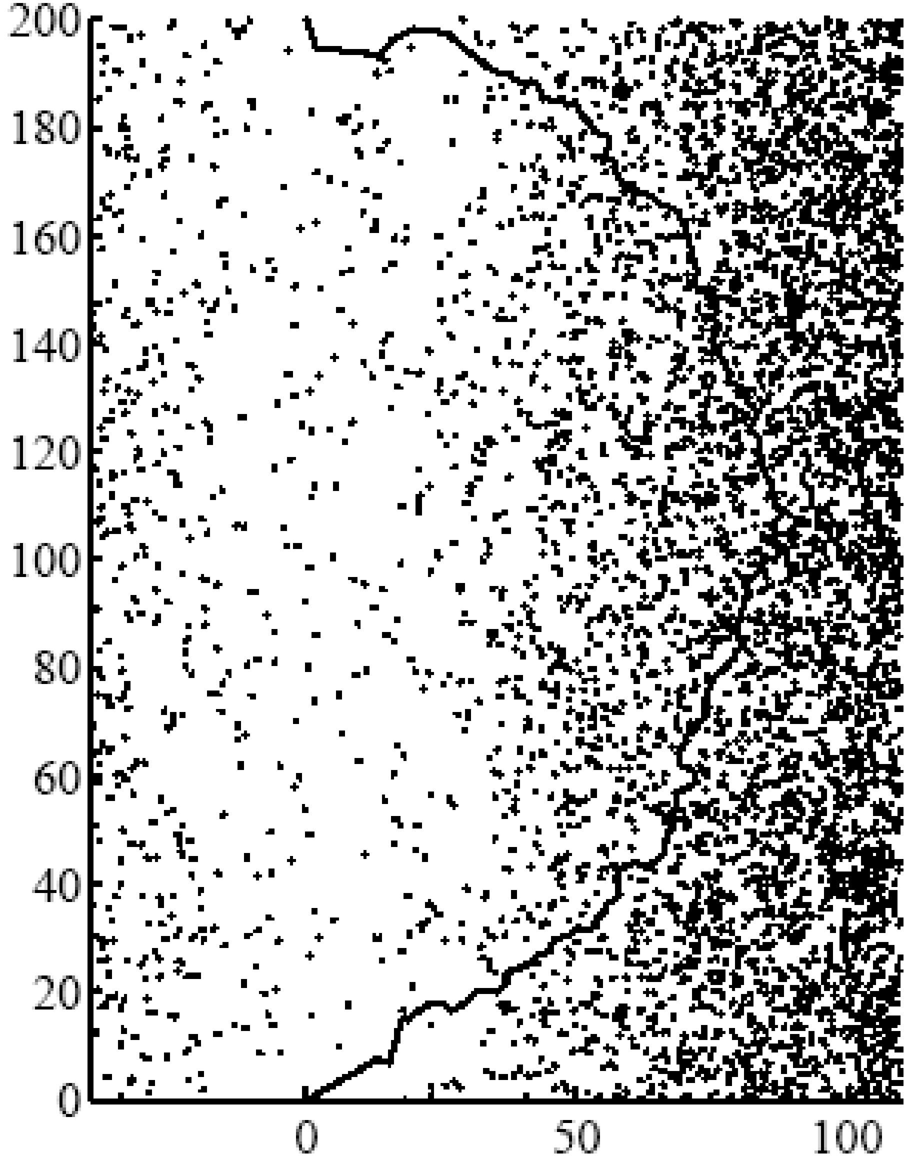
$$\lambda(x, y) = \frac{1}{10}\left(10^{-4} x^2 + 0.05\right)$$



$$\lambda(x,y)=\frac{1}{5}\left(10^{-4}x^{2}+0.05\right)$$

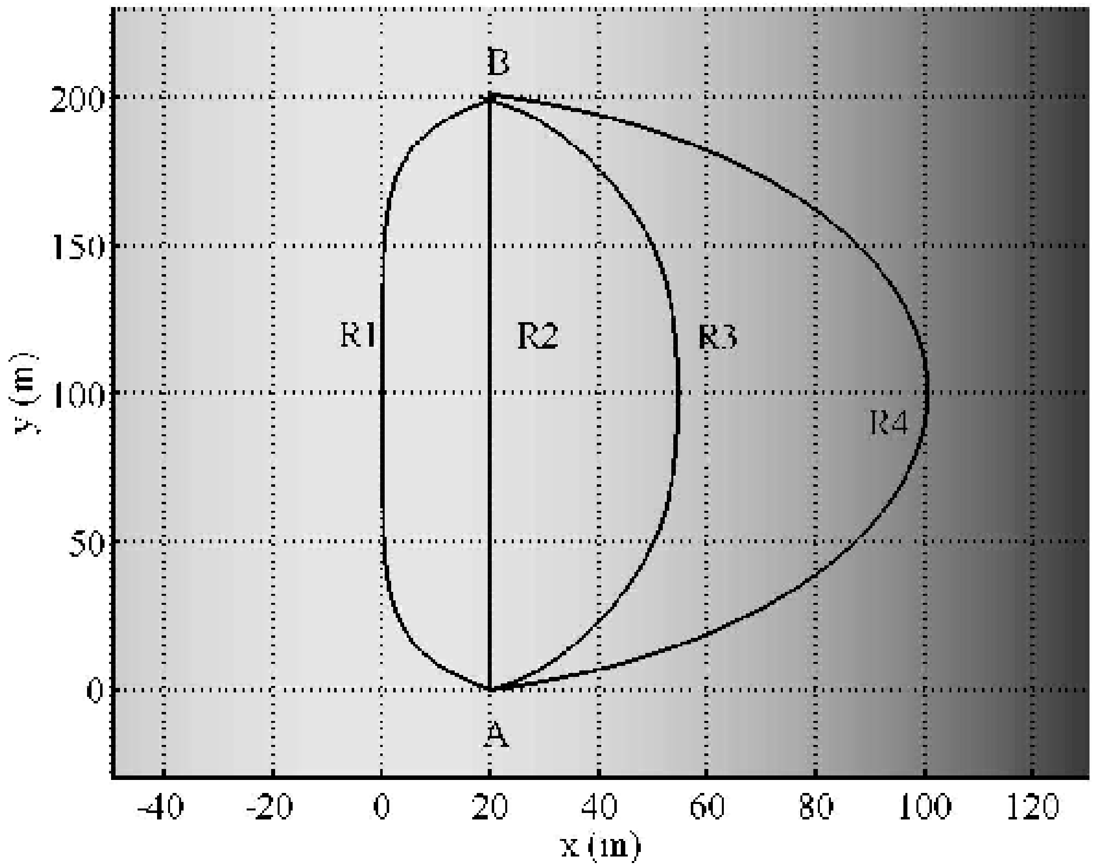


$$\lambda(x,y) = \frac{1}{2}\left(10^{-4}x^2 + 0.05\right)$$



- Question: what happens in the limit?

Limiting case predicted by Optics



P. Jacquet, “Geometry of Information Propagation in massively dense ad hoc networks,” MOBIHOC 2004

Macroscopic formulation

- Cost Function: $c(\mathbf{r}) = \lim_{\varepsilon \to 0} \frac{dc(\varepsilon, \mathbf{r})}{\varepsilon}.$

- Cost of route *C* that starts at *A* and ends at *B*:

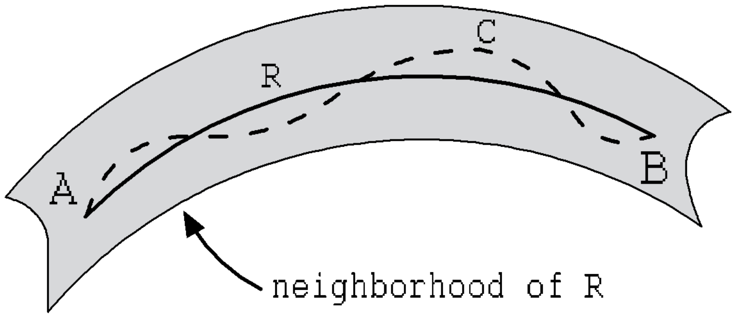
$$[AB]_C = \int_A^B c(\mathbf{r}) d\mathbf{r}.$$

- Problem: Find route from *A* to *B* that minimizes cost.

Relation to Optics

- Fermat's Principle: To travel from A to B, light will take the route that ***locally*** minimizes the integral:

$$\int_A^B dt = \int_A^B \frac{1}{u} ds = \frac{1}{c} \int_A^B n(\mathbf{r}) ds.$$



- Therefore we have the following analogy:
  - Index of refraction $n(\mathbf{r})$ becomes the cost function $c(\mathbf{r})$
  - Rays of light become minimum-cost routes.

The advantages of Optical Routing

- We can use the rich body of math that already exists in Optics for our setting.
  - For example, we know that light satisfies the following equations:

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n, \qquad |\nabla S| = n.$$

- We can use the intuition that already exists.
  - For example, we know that rays of light bend toward optically denser materials.

Various Choices for the Cost Function

1. Promoting long hops $c(\mathbf{r}) = \sqrt{\lambda(\mathbf{r})}$

2. Promoting short hops $c(\mathbf{r}) = \frac{1}{\sqrt{\lambda(\mathbf{r})}}$

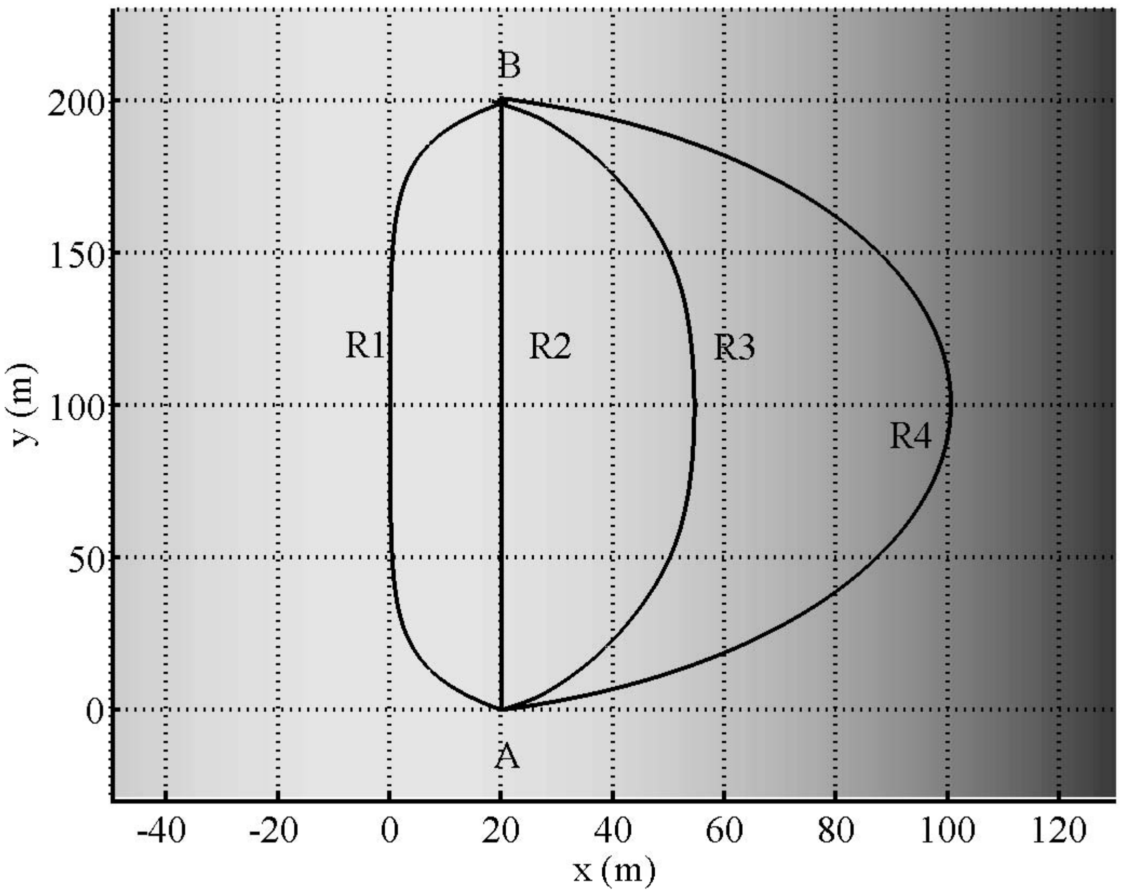
3. Promoting energy efficiency

$$c(\mathbf{r}) = f(\lambda(\mathbf{r})),$$

$$f(x) \to \infty, x \to 0, \qquad f(x) \to const, x \to \infty.$$

4. Etc.

Choice of cost function very important!

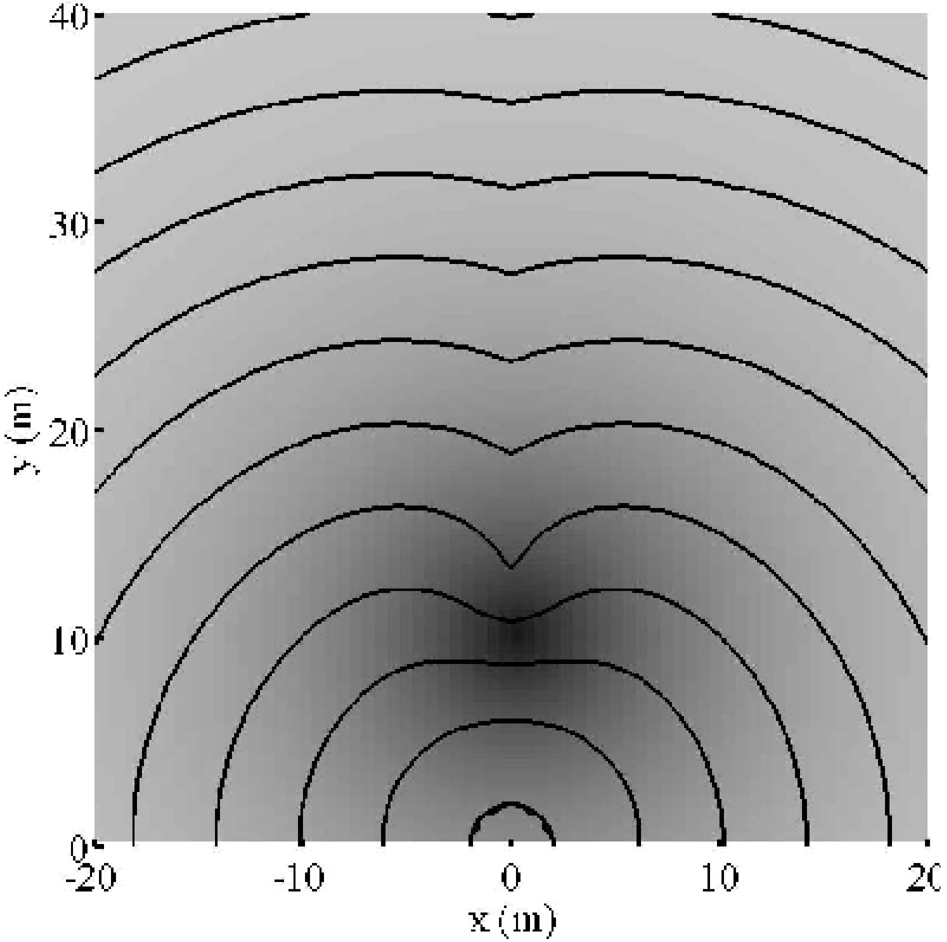


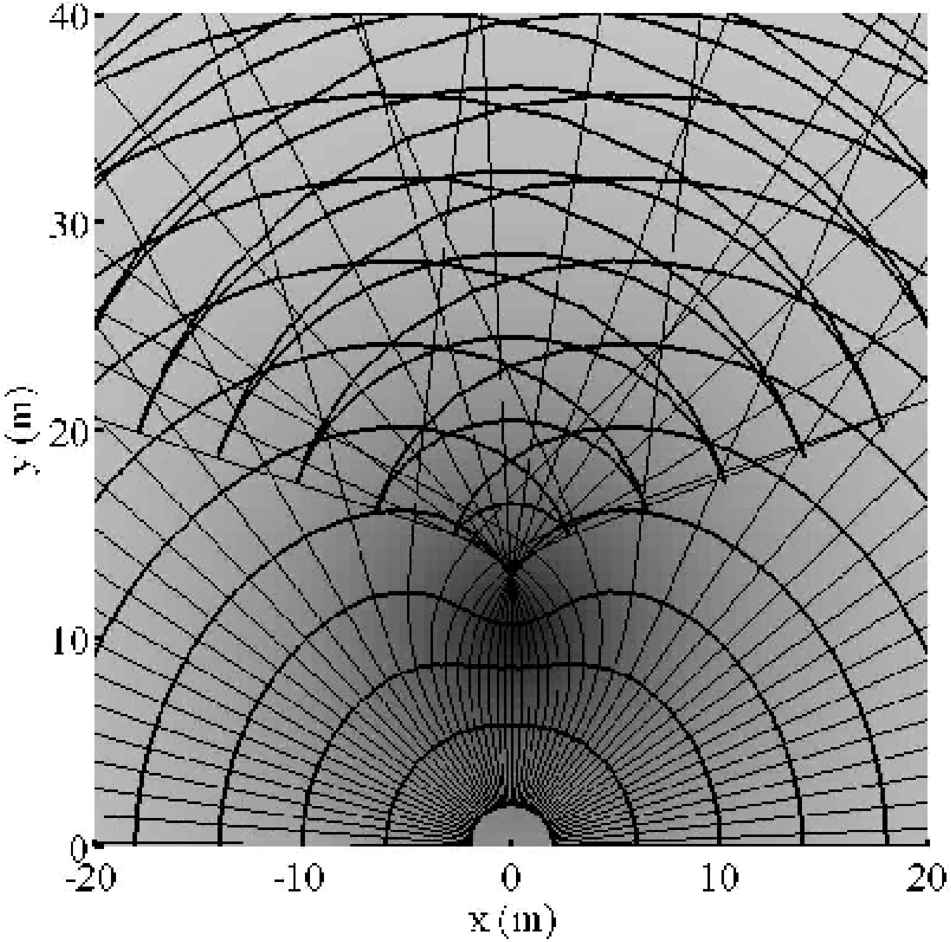
R1: Jacquet, R2: Constant cost,

R3: Energy limited, R4: Bandwidth limited

Broadcast Routing

The optimal propagation of a packet resembles the propagation of light emanating from a light source

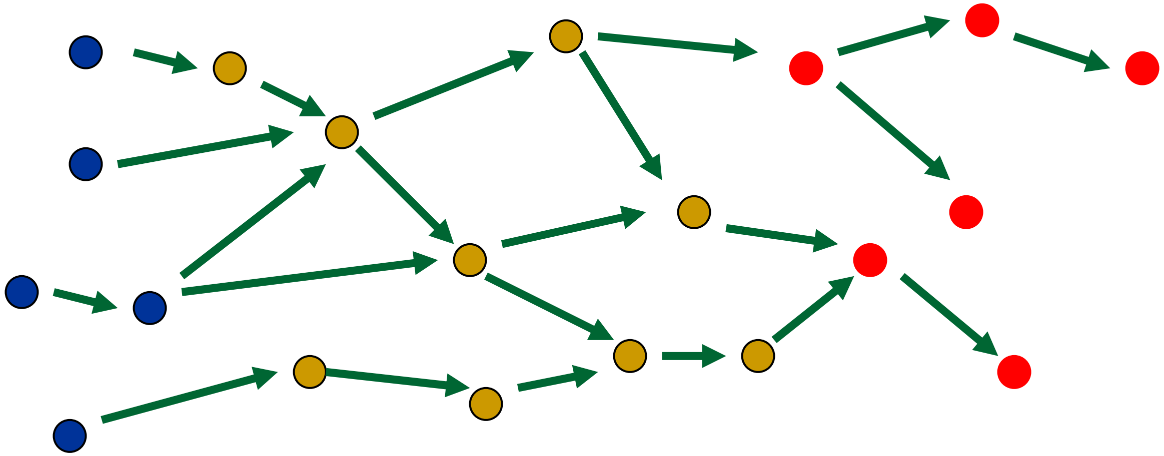




Part C: “Interferistors”

S. Toumpis and S. Gitzenis, “Load Balancing in Wireless Sensor Networks using Kirchhoff’s Voltage Law,” in *Proc. IEEE Infocom 2009*, Rio de Janeiro, Brazil., Apr. 2009.

Motivation: Wireless Sensor Networks



- Sources send traffic to Sinks by way of Relays using wireless links
- Wireless links interfere with each other
- Problem: minimize the total cost of the data flow

(Note: our work applies to all wireless single commodity problems, but makes particular sense for WSNs)

Our Optimization Problem

$$\text{minimize} \quad F(\mathbf{x}) = \sum_{(i,j)\in A} f_{ij}(x_{ij}; \mathbf{x}_{ij})$$

$$\text{subject to} \quad s_i = \sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji}$$

1. A is the set of all arcs

2. Flow through arc (i,j)

3. Vector of all flows

4. Set of flows inter-fering arc (i,j)

5. Arc cost incurred at arc (i,j)

6. Total Cost

7. Data injection rate in node i:
   * If $s_i>0$, i is a source
   * If $s_i<0$, i is a sink
   * If $s_i=0$, i is a plain relay

8. What comes out of node i

9. What goes in node i

Kirchhoff's Voltage Law (KVL)

- For separable cost (i.e., no interference), optimality condition ⇔ KVL if we substitute links with electrical elements of *V*-*I* characteristic

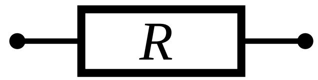
$$V_{ij}(I_{ij}) = \frac{\partial f_{ij}(I_{ij})}{\partial x_{ij}}$$

- In the general case, optimality optimality condition ⇔ KVL if we substitute links with **interferistors**
- Hence, intuition and methods of Circuit Theory can be applied here

Examples (1/2)

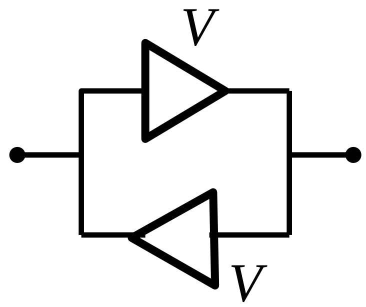
1. Resistor:

$$f_{ij}(x_{ij}) = \frac{R}{2}x_{ij}^2 \Leftrightarrow \frac{\partial f_{ij}(x_{ij})}{\partial x_{ij}} = Rx_{ij} \leftrightarrow V_{ij}(I_{ij}) = RI_{ij}$$



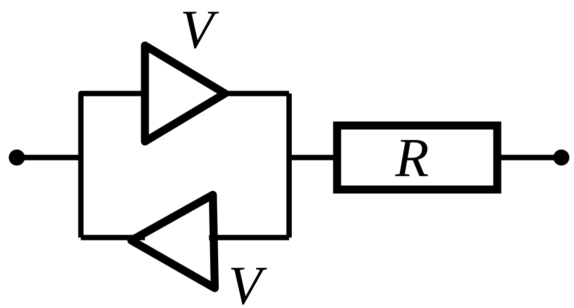
2. Parallel diodes:

$$f_{ij}(x_{ij}) = V\left|x_{ij}\right| \Leftrightarrow \frac{\partial f_{ij}(x_{ij})}{\partial x_{ij}} = \begin{cases} V, & x_{ij} > 0 \\ -V & x_{ij} < 0 \end{cases} \leftrightarrow V_{ij}(I_{ij}) = \begin{cases} V, & I_{ij} > 0 \\ -V & I_{ij} < 0 \end{cases}$$



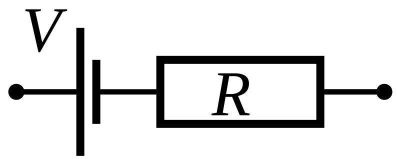
3. Parallel diodes and resistor:

$$f_{ij}(x_{ij}) = V\left|x_{ij}\right| + \frac{R}{2}x_{ij}^2 \leftrightarrow V_{ij}(I_{ij}) = \begin{cases} RI_{ij} + V, & I_{ij} > 0 \\ RI_{ij} - V & I_{ij} < 0 \end{cases}$$



Examples (2/2)

4. Voltage source and resistor:

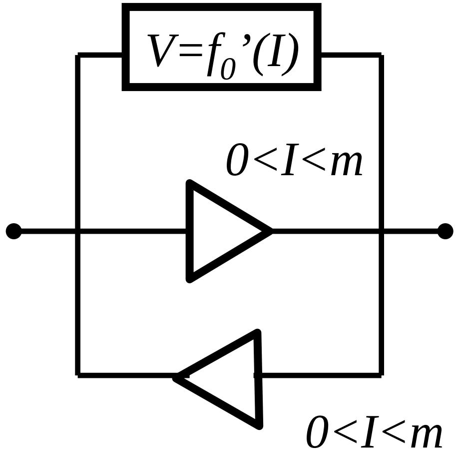


$$f_{ij}(x_{ij}) = Vx_{ij} + \frac{R}{2}x_{ij}^2 \Leftrightarrow \frac{\partial f_{ij}}{\partial x_{ij}} = V + Rx_{ij} \leftrightarrow V_{ij}(I_{ij}) = V + RI_{ij}$$

4. Current limiting diodes:

$$f_{ij}(x_{ij}) = \begin{cases} f_0(x_{ij} - m), & x_{ij} > m \\ f_0(0), & -m < x_{ij} < m \\ f_0(x_{ij} + m), & x_{ij} < -m \end{cases}$$

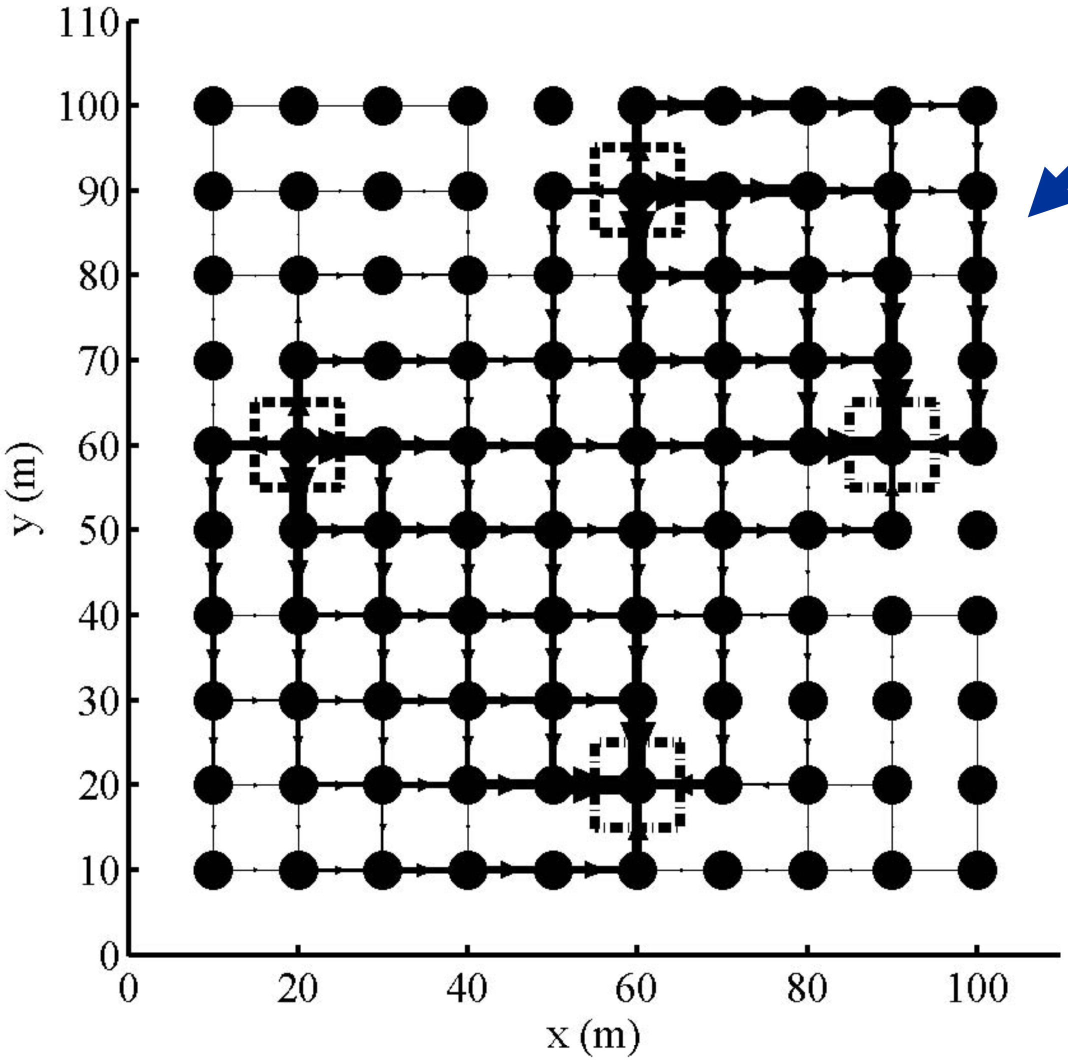
$$\leftrightarrow V_{ij}(I_{ij}) = \begin{cases} f_0^{'}(I_{ij} - m), & I_{ij} > m \\ f_0(0), & -m < I_{ij} < m \\ f_0^{'}(I_{ij} + m), & I_{ij} < -m \end{cases}$$

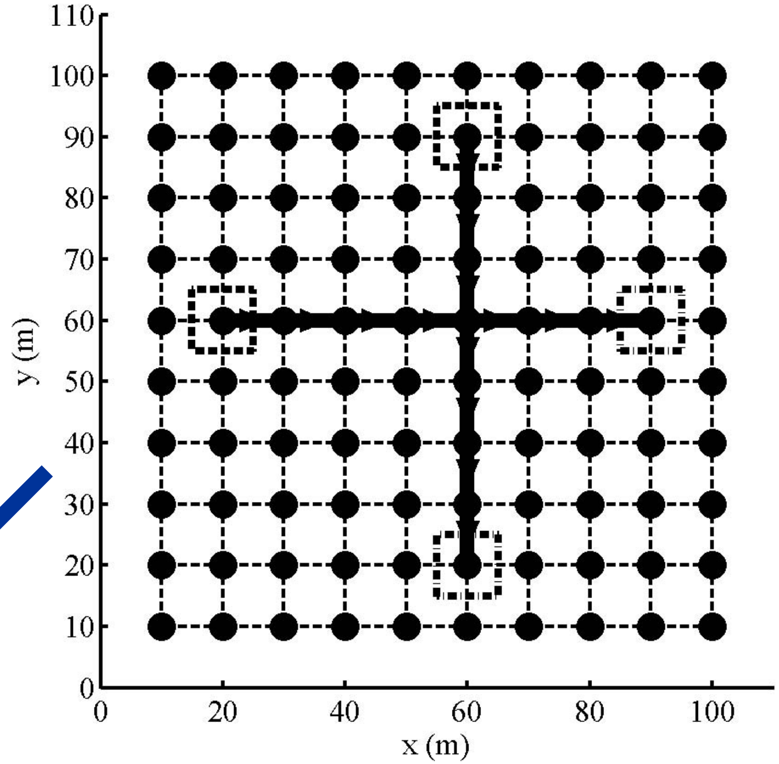


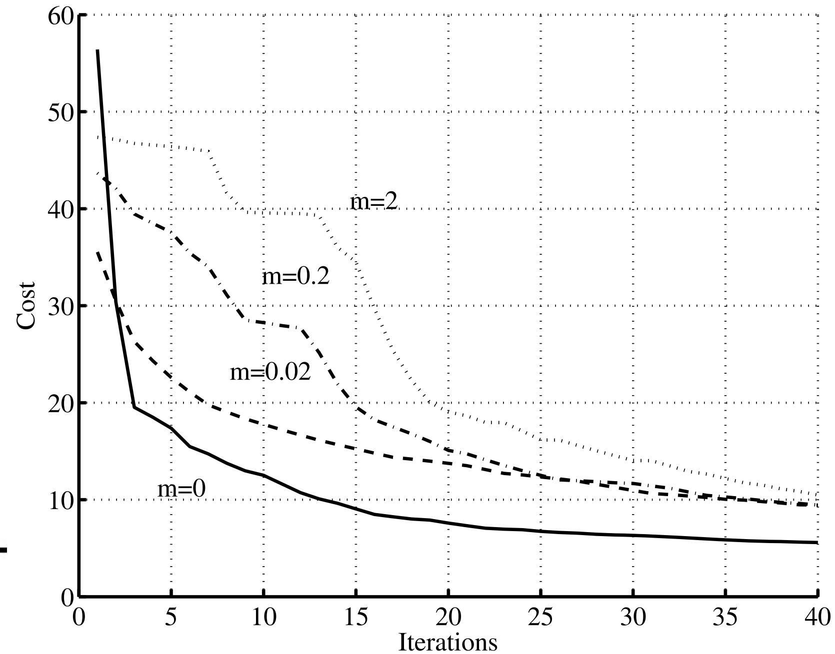
Primal Algorithm

- **Initialization:** Form a *sufficient* set of cycles in the network
  - No holes: only local operations
  - Holes: minimal global algorithms are needed
- **Main Loop:** Improve the overall cost by introducing circulations along these cycles
  - Intuition: try to satisfy the KLV is in those loops
  - Guaranteed to converge to (local) minimum

Example: Network with *Interferistors*







Parting Comments

- Analogies with Physics are well worth investigating
- The field is particularly promising in wireless networks due to their spatial aspect
- I showed you three examples of such analogies
- Can you come up with others, in your own research?

BACKUP SLIDES

Applications

- Automatic exchange of sensor data between cars
  - emergency braking, slippery road
- Transmission of information useful to driver
  - “traffic jam ahead”
  - “road works ahead”
  - weather reports
  - software upgrades, etc.
- Internet Access in the Car (“Infotainment”)
- Transmission of Advertisements
  - e.g. you are approaching a McDonalds

What goes in, must come out

- The net amount of information leaving a surface A_0 through its boundary $B(A_0)$, must be equal to the net amount of information created in that surface:

$$\oint_{B(A_0)} \left[\mathbf{T} \cdot \hat{\mathbf{n}}(s)\right] ds = \int_{A_0} \rho(x, y) dS$$

- Taking $|A_0| \rightarrow 0$, we get the requirement:

$$\nabla \cdot \mathbf{T} \equiv \frac{\partial \mathbf{T}_x}{\partial x} + \frac{\partial \mathbf{T}_y}{\partial y} = \rho \quad (1)$$

Special Case

1. Nodes only need to transfer data from sources to sinks
   1. They do not need to sense them at the sources
   2. They do not need to deliver them to the sinks once their location is reached
2. The traffic flow function and the node density function are related by:

$$|\mathbf{T}(x, y)|_{\max} = c\sqrt{d(x, y)} \qquad (2)$$

⇒Traffic must be irrotational

- We must minimize the number of nodes

$$N = \int d(x, y) dA.$$

- If (2) is satisfied, then the traffic must be irrotational:

$$\nabla \times \mathbf{T} \equiv \frac{\partial \mathbf{T}_y}{\partial x} - \frac{\partial \mathbf{T}_x}{\partial y} = 0.$$

- Easy proof by contradiction.

A final look at the optimization problem

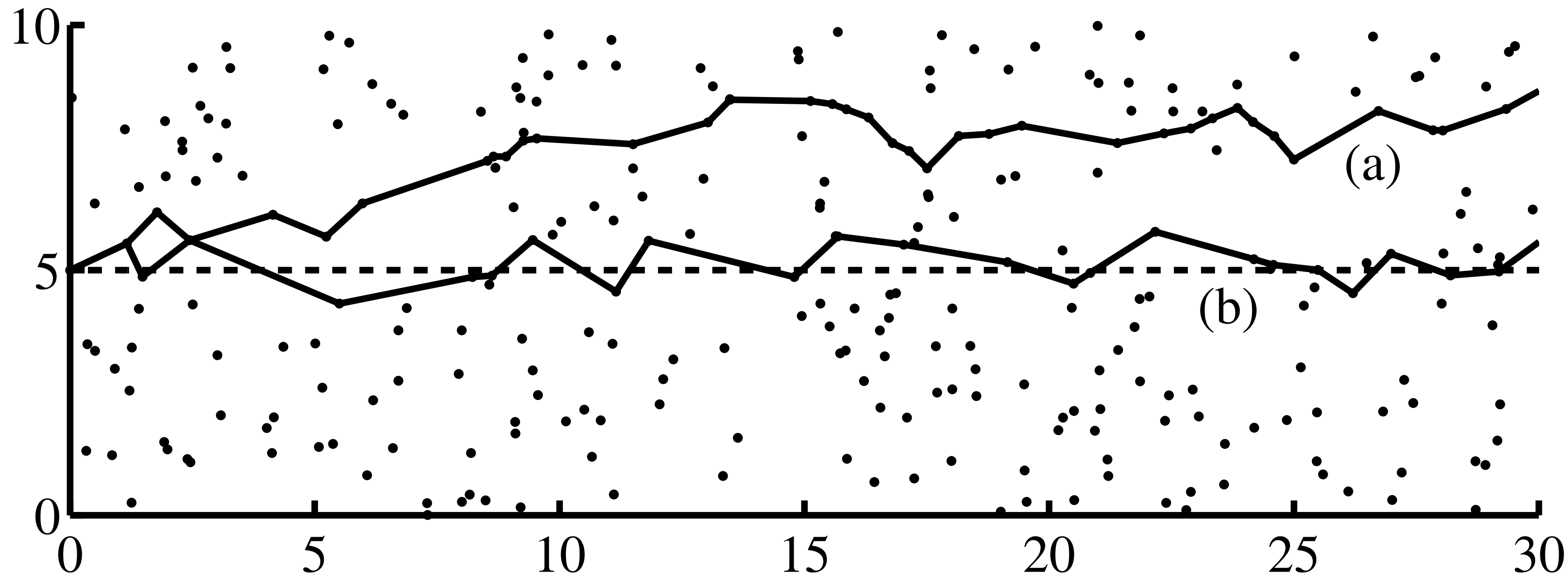
$$\text{minimize:} \qquad N = \int G(x, y, |\mathbf{T}(x, y)|^2) dS,$$

$$\text{subject to:} \qquad \nabla \cdot \mathbf{T}(x, y) = \rho(x, y).$$

- The integrant can have alternative interpretations: delay, energy, etc.
- This is a problem in optimal transportation

Any practical gain by knowing the limit?

- With finite but many nodes, the optimum route is hard to find
- So let us find the optimum route in the macroscopic limit, and use it to create a near optimum route

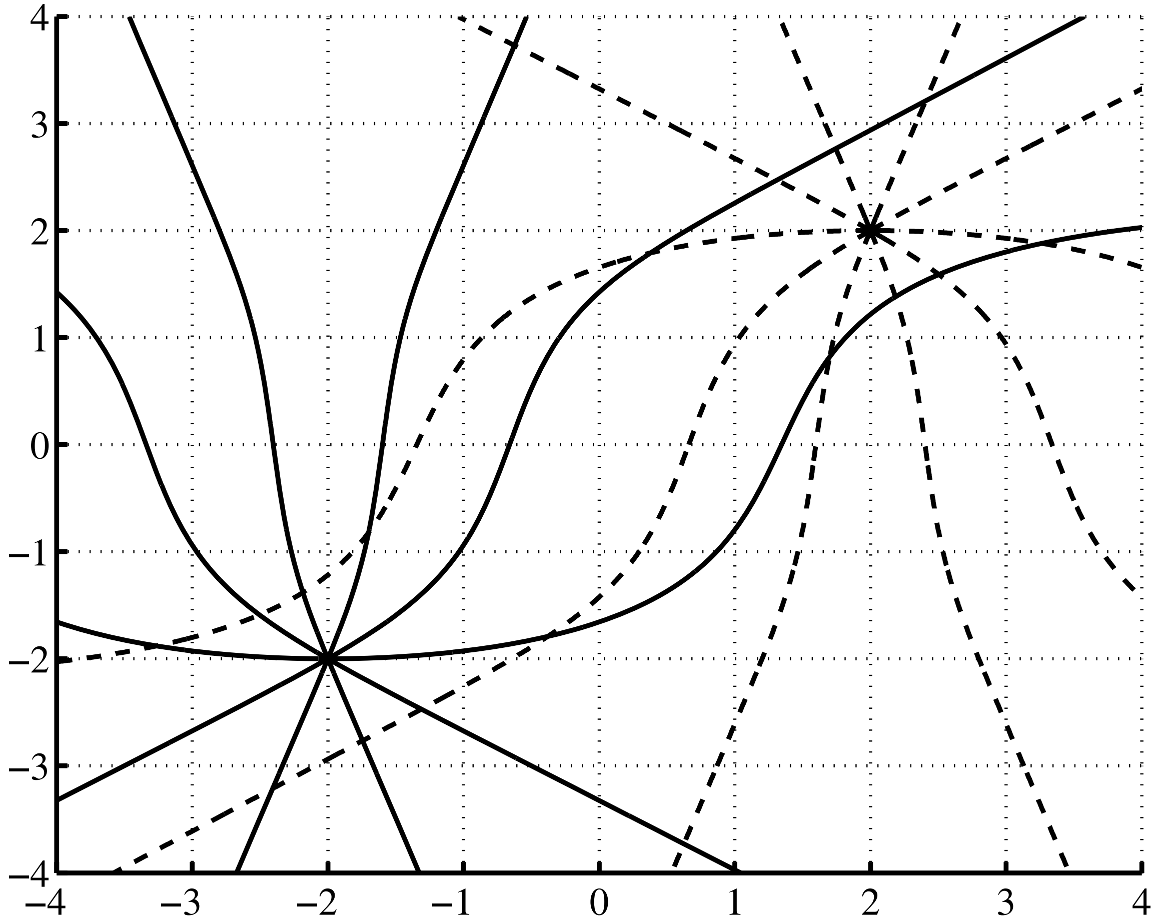


What the Optics-Networking Analogy does not tell us

- How does the source know the initial angle with which the packet/ray should be launched?

- In some nonhomogeneous environments, there are multiple rays connecting two points
  - All of them local minimums
  - One of them global minimum

Route Discovery



- Basic idea: Nodes launch multiple rays
- Intersection points notify pairs of node

An Example Arc Cost

6. $n=0$: approximates queueing delay in arc
   $n=1$: approximates average queue length
   $n=2$: promotes load balancing

1. Cost at arc (i,j)

2. Own flow

$$f_{ij}(x_{ij};\mathbf{x}_{ij}) = \frac{|x_{ij}|^n}{U_{ij} - |x_{ij}| - \sum_{x_{ij} \in \mathbf{x}_{ij}} |x_{kl}|}$$

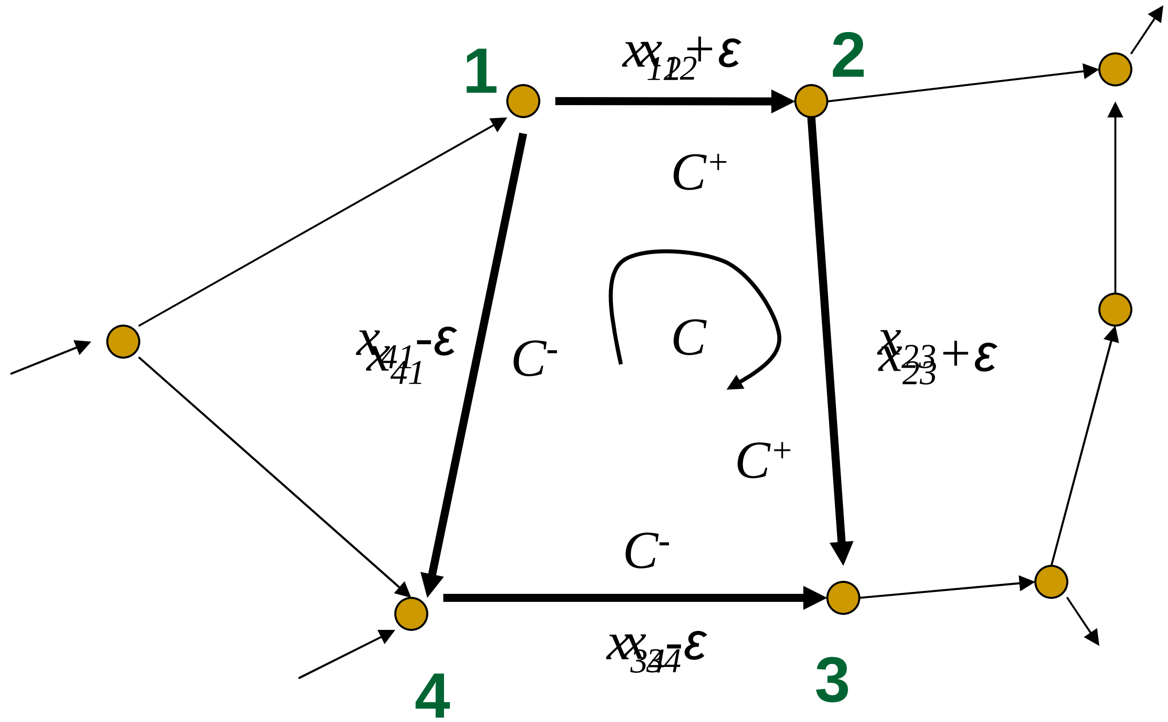
4. Flows at interfering arcs

3. Arc (i,j) capacity

5. Interfering arcs subtract from the available capacity

(Obviously, many other choices are possible, depending on the setting. Convex choices are desirable.)

Necessary Condition



- Let **x** be a flow of traffic
- If I can do better by introducing a small circulation along a cycle, then **x** cannot be (locally) optimal

Necessary Condition, Formally

- **Proposition 1:** Let $F(\mathbf{x})$ be continuously differentiable. If the traffic flow vector $\mathbf{x}^*$ is (locally) optimal, then if we introduce a small circulation along any cycle C, the incremental cost is zero:

$$\sum_{(i,j)\in C^+} \frac{\partial F(x^*)}{\partial x_{ij}} - \sum_{(i,j)\in C^-} \frac{\partial F(x^*)}{\partial x_{ij}} = 0. \quad \text{(A)}$$

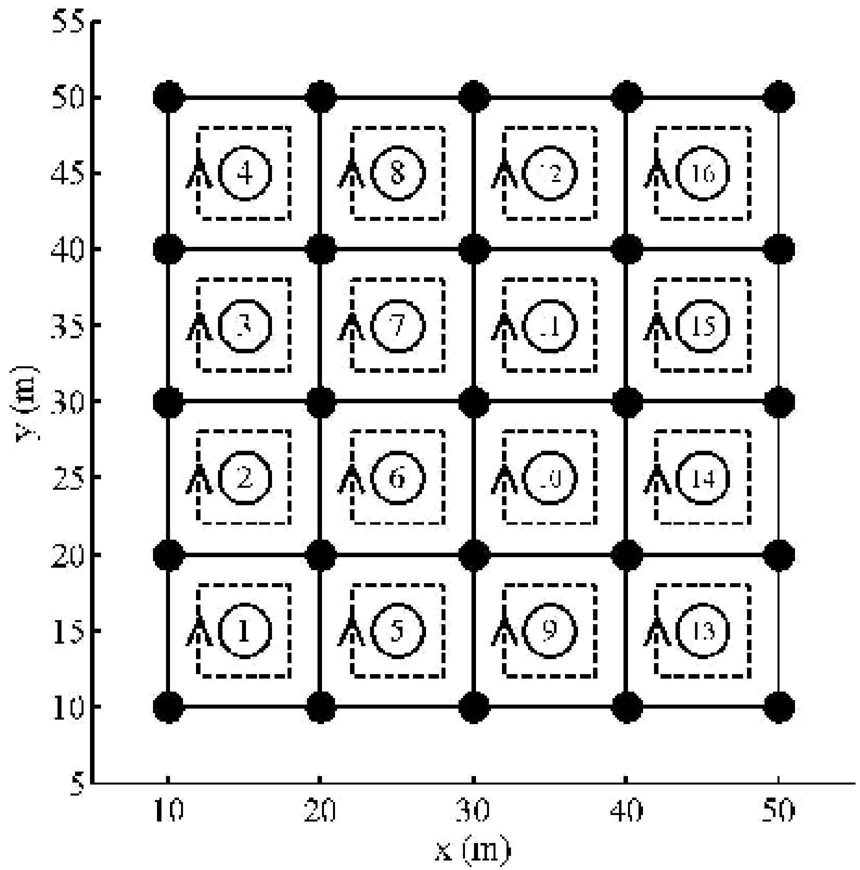
All arcs in the forward direction of C

Incremental total cost of increasing flow in (i,j)

All arcs in the backward direction of C

Sufficient Condition

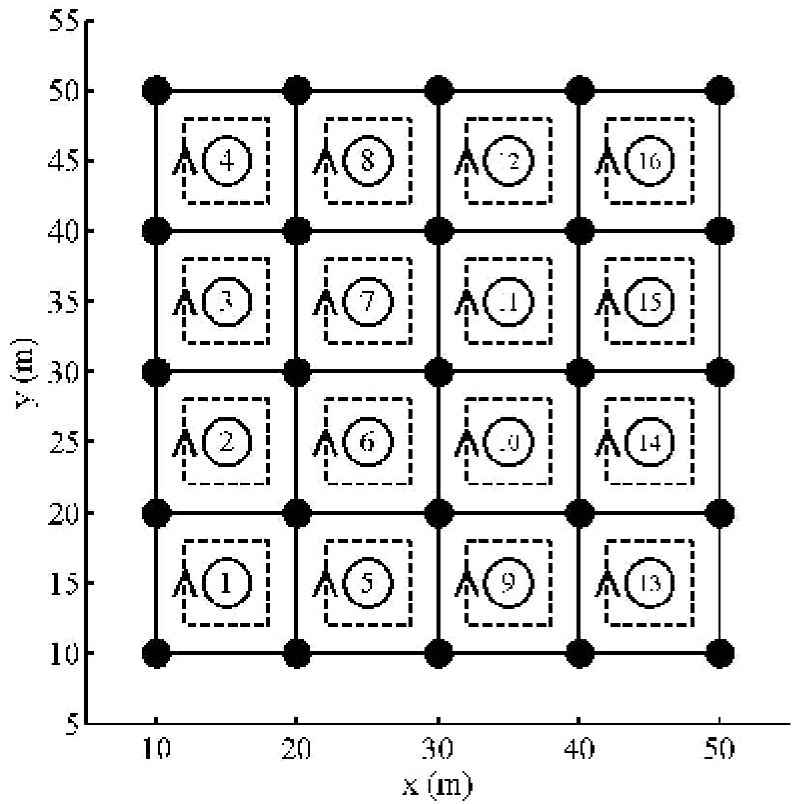
- If **x** <u>cannot</u> be improved by introducing a small circulation along <u>any</u> cycle, then **x** <u>must</u> be optimal.
- No need to check <u>all</u> cycles. A minimal set should exist.



First, a Definition

- Let $Z(\mathcal{G})$ be the **circulation space** of the graph $\mathcal{G}$ of the network, i.e., the set of all circulations.

In this example, $Z(\mathcal{G})$ is 16-dimensional,
and one choice for $\mathcal{C}$ is
$\mathcal{C}=\{1,2,\ldots,16\}$



Sufficient Condition, Formally

- **Proposition 3:** Let $F(\mathbf{x})$ be continuously differentiable. If Condition (A) holds for all cycles in a set $\mathcal{C}$ that spans the circulation space, then
  - $\mathbf{x}$ is a stationary point (if $F(\mathbf{x})$ not convex)
  - or globally optimal (if $F(\mathbf{x})$ convex)

(Note: theory extends for practically any $F(\mathbf{x})$ an engineer might want to use – see Props. 2 and 4 of the paper.)

Overhead

1. Message Passing:
   - Arcs must send information about their levels of congestion to all arcs affecting them
   - For most cases the information is single number and can be piggybacked
- Cycle Maintenance:
  - Each node must maintain cycles in neighborhood
  - Each node must be able to identify when a cycle can break down in two smaller ones
  - Some nodes must be able to spot network-wide holes
  - Only need span, not basis

Comments on the Algorithm

- The simplex algorithm and various algorithms from circuit theory (method of nodes, method of meshes, etc.) also are based on cycles, but:
  - They are centralized and
  - The cost function is separable
- Our basic element is the **cycle**
  - NUM is based on the **links**
  - Other algorithms are based on **end-to-end paths**
- Most of the complexity lies in the initialization phase

Wireless Sensor Technology


UC Berkeley: COTS Dust


UC Berkeley: COTS Dust

UC Berkeley: Smart Dust


UCLA: WINS


Rockwell: WINS


JPL: Sensor Webs